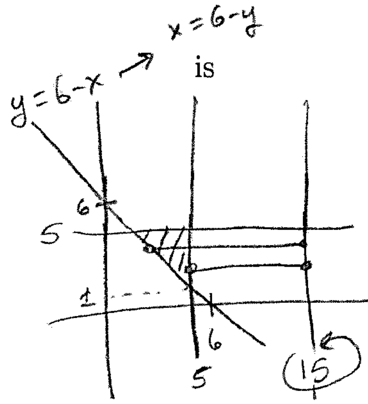
QUIZ 13 SOLUTIONS: LESSONS 14-15
FEBRUARY 20, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] The integral that represents the volume of the solid generated by revolving the given region about the line $x = 15$:

$$y = 6 - x,\ y = 5,\ \text{and}\ x = 5$$

is



$$\int_1^5 \pi\left[(15-(6-y))^2 - (15-5)^2\right] dy$$

$$= \boxed{\int_1^5 \pi\left[(9+y)^2 - (10)^2\right] dy}$$

2. [5 pts] Determine whether

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

converges or diverges. If it converges, find its value. Round to 4 decimal places.

HINT: Use u-substitution.

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx := \lim_{t\to\infty} \int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$$

$u = -\sqrt{x}$ $\qquad u(t) = -t^{1/2}$

$du = \frac{-1}{2\sqrt{x}}\,dx$ $\qquad u(1) = -\sqrt{1} = -1$

$$\int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \int_{-1}^{-t^{1/2}} \frac{1}{\sqrt{x}} e^u(-2\sqrt{x})\,du$$

$$= \int_{-1}^{-t^{1/2}} -2e^u\,du$$

$$= -2e^u \Big|_{-1}^{-t^{1/2}}$$

$$= -2e^{-t^{1/2}} - (-2e^{-1})$$

$$= \frac{-2}{e^{t^{1/2}}} + \frac{2}{e}$$

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty}\left[-\frac{2}{e^{t^{1/2}}} + \frac{2}{e}\right]$$

$$= \frac{2}{e}$$

$$\approx \boxed{.7358}$$